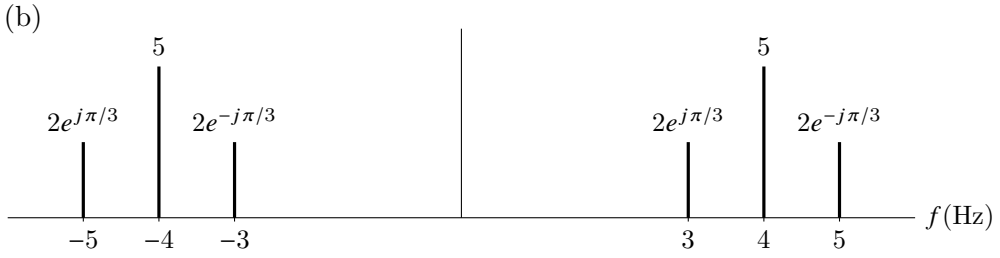
CHAPTER 3

연속시간 푸리에급수

P3.1 (a) $x_{AM}(t) = 10\cos(2\pi(1000)t) + 4\cos\left(2\pi(900)t + \frac{\pi}{3}\right) + 4\cos\left(2\pi(1100)t - \frac{\pi}{3}\right)$

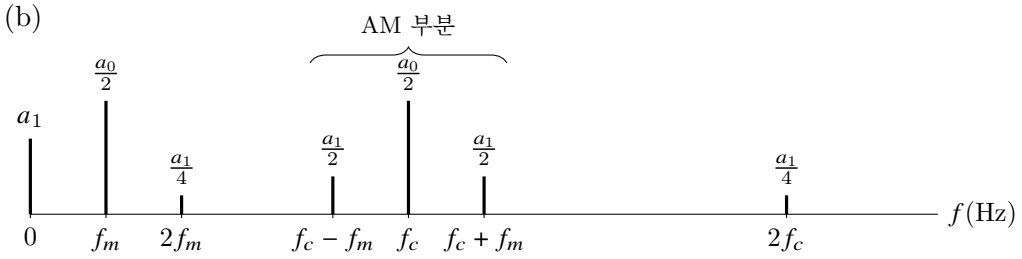
(b)



P3.2 (a) $y(t) = \underbrace{a_0[1 + \frac{2a_1}{a_0}\cos(2\pi f_m t)]\cos(2\pi f_c t)}_{AM\text{부분}}$

$$+ \frac{a_1}{2}\cos(2\pi(2f_m)t) + \frac{a_1}{2}\cos(2\pi(2f_c)t) + a_0\cos(2\pi f_m t) + a_1$$

(b)



P3.4 $x(t) = 4\cos(\pi t - \pi/4) + 2\sin(3\pi t)$

P3.5 (a) 기본주파수 $f_0 = 10\,\text{Hz}$, 기본 주기 $T_0 = 0.1\,\text{sec}$

(b) $a_0 = 1, \quad a_2 = \frac{3}{2}e^{j\pi/4} = a_{-2}^*, \quad a_4 = \frac{5}{2} = a_{-4}, \quad a_5 = 4e^{j\pi/3} = a_{-5}^*$

(c) $b_0 = a_0 = 1, \quad b_4 = a_2 = \frac{3}{2}e^{j\pi/4} = b_{-4}^*, \quad b_8 = a_4 = \frac{5}{2} = b_{-8}, \quad b_{10} = a_5 = 4e^{j\pi/3} = b_{-10}^*$

$b_5 = \frac{1}{2} = b_{-5}$

P3.6 (a) 기본주파수 $f_0 = 50\,\text{Hz}$, 기본 주기 $T_0 = 2\,\text{msec}$

(b) $a_1 = e^{-j\pi/2} = a_{-1}^*, \quad a_4 = 5e^{-j\pi/3} = a_{-4}^*, \quad a_6 = 2e^{j\pi/4} = a_{-6}^*$

(c) $e^{j2\pi(kf_0)t}$ 의 직교성을 이용

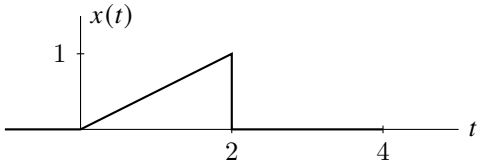
(d) $b_2 = a_4 = b_{-2}^*, \quad b_3 = a_6 = b_{-3}^*$

P3.8 (a) $a_0 = \frac{1}{2}, \qquad a_k = \frac{1}{jk2\pi}\left(1 - e^{-jk8\pi/5}\right)$

(b) $a_0 = \frac{1}{2}, \qquad a_k = \frac{25}{4}\left(\frac{1}{jk2\pi}\right)^2\left[1 - e^{-jk8\pi/5}\right]$

P3.9 $a_k = \frac{1}{1 + jk2\pi}[1 - e^{-1}]$

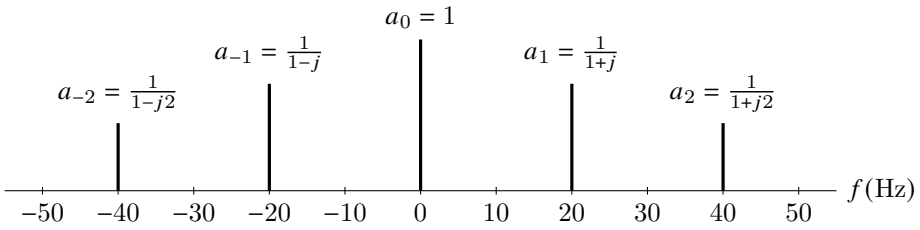
P3.10



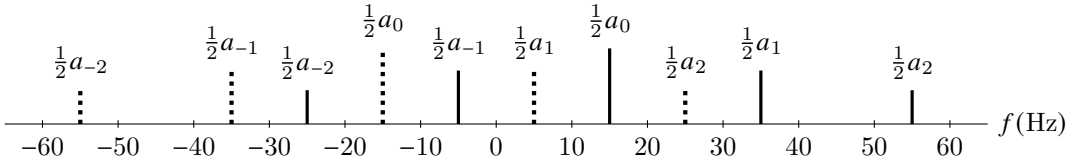
P3.11 (a) 신호 $x(t)$의 기본 주파수는 20 Hz이며, 푸리에급수 계수는

$$a_0 = 1, \quad a_1 = \frac{1}{1+j} = a_{-1}^*, \quad a_2 = \frac{1}{1+j2} = a_{-2}^*$$

(b)



(c) $$y(t) = \underbrace{\frac{1}{2}\sum_{k=-2}^{2}\left(\frac{1}{1+jk}\right)e^{j2\pi(20k+15)t}}_{y_1} + \underbrace{\frac{1}{2}\sum_{k=-2}^{2}\left(\frac{1}{1+jk}\right)e^{j2\pi(20k-15)t}}_{y_2}$$



(d) 기본 주파수가 5 Hz. $y(t)$의 푸리에급수 계수 b_k는 다음과 같다.

$$b_1 = \frac{1}{2}a_1, \quad b_3 = \frac{1}{2}a_0, \quad b_5 = \frac{1}{2}a_2, \quad b_7 = \frac{1}{2}a_1, \quad b_{11} = \frac{1}{a_2}$$

여기서 $a_{-k} = a_k^*$ 이므로 $b_k = b_k^*$.

P3.12 (a) 기본 주파수는 $f_0 = \text{최대공약수}(50, 90) = 10\,\text{Hz}$

(b) $a_5 = 2 + 3j = \sqrt{13}e^{j\theta} = a_{-5}^*, \quad \theta = \arctan\left(\frac{3}{2}\right) \approx 0.983$

$a_9 = 2j = 2e^{j\pi/2} = a_{-9}^*$

(c) $x(t) = 2\sqrt{13}\cos(2\pi(50)t + 0.983) + 4\cos(2\pi(90)t + \pi/2)$

(d) $b_5 = -300\pi + j200\pi = b_{-5}$

$b_9 = (j180\pi)(2j) = -360\pi$

(e) $c_5 = -j(2 + j3) = 3 - j2 = c_{-5}$

$c_9 = e^{-j0.9\pi}(2j) = 2e^{-j0.4\pi}$

P3.13 (a)

$$a_0 = 1, \qquad a_k = \begin{cases} -\dfrac{2}{j\pi k}, & \text{짝수 } k \\ -\dfrac{2}{j\pi k} - \dfrac{4}{(j\pi k)^2}, & \text{홀수 } k \end{cases}$$

(b)

$$b_0 = -2 \qquad b_k = \begin{cases} \dfrac{4}{jk\pi}, & \text{짝수 } k \\ 0, & \text{홀수 } k \end{cases}$$

P3.14 (a)

$$a_k = \begin{cases} 0, & k = 0 \\ \dfrac{1}{-j\pi k}, & k \neq 0 \end{cases}$$

(b)

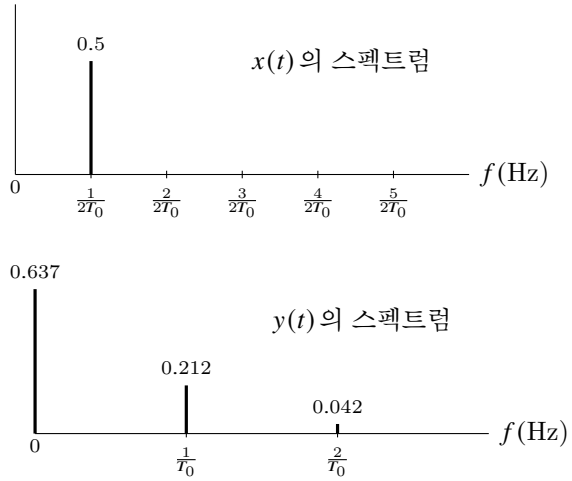
$$a_k = \begin{cases} 0.5, & k = 0 \\ \dfrac{1}{2j\pi k}[1 - (-1)^k], & k \neq 0 \end{cases}$$

P3.17 (a)

$$P_y = \frac{1}{T_0}\int_0^{T_0} |\sin(\pi t/T_0)|^2 dt = \frac{1}{2}$$

(b) $b_k = \dfrac{1}{\pi(1-2k)} + \dfrac{1}{\pi(1+2k)} = \dfrac{2}{\pi(1-4k^2)}$

(c)



(d) $y(t)$ 하모닉 성분 각각의 평균전력과 누적합은 다음과 같다.

$$\text{DC 성분,}\quad |b_0|^2 = (0.637)^2 \approx 0.406$$

$$\text{첫 번째 하모닉,}\quad 2|b_1|^2 = 2(0.212)^2 \approx 0.09 \quad\Longrightarrow\quad 0.406 + 0.09 = 0.496$$

$$\text{두 번째 하모닉,}\quad 2|b_2|^2 = 2(0.042)^2 \approx 0.0035 \quad\Longrightarrow\quad 0.496 + 0.0035 = 0.4985$$

하모닉 스펙트럼 평균전력의 누적합은 두 번째 성분에서 이미 전체 평균전력 0.5의 99.7 %이다.

P3.18 $$Q = \sum_{l=0}^{\infty} \left(\frac{1}{2l+1}\right)^4 = \frac{\pi^4}{96}$$

P3.19 신호에 불연속점이 있으므로 깁스현상 발생한다.